RESEARCH BULLETIN

A FACTOR ANALYSIS OF THE PICTURE COMPLETION ITEMS OF THE WAIS

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Items of the WAIS

Abstract

Factor analysis is applied to a matrix of item intercorrelations for items of the WAIS Picture Completion subtest, using a sample of 228 college-oriented males. Three factors are found, and identified in terms of the three principal causes of failure on PC described by Rapaport. The factors are examined in relation to the overall framework of the WAIS reported by Saunders, and found not to require any new dimension. The multidimensional character of PC explains why the items of this subtest have been particularly resistant to scaling for difficulty.

Factor I is most appropriately labelled "Maintenance of Contact" versus "Loss of Contact" or withdrawal, is best measured by items 16 and 18, and is not linearly related to any other subtest. This factor, which apparently embodies the unique contribution of PC to the Wechsler battery, might also be called "Recognition of Incongruity."

Factor II is most appropriately labelled "Maintenance of Perspective" versus "Loss of Distance," is best measured by items 8 and 9, and is related to certain items in the Comprehension subtest.

Factor III is related to the effect of "uncertainty" on the subject, is best measured by items 2 and 14, and is related to total scores on the Object Assembly subtest.

A Factor Analysis of the Picture Completion Items of the WAIS[1,2]

As a result of a recently reported factor analytic study of the WAIS (6) it appears very likely that there are at least as many statistically significant dimensions tapped by the battery as there are distinct subtests, but that the factor structure and the subtest structure of the battery are _not_ congruent. In particular, two of the eleven factors obtained in that study could neither be discarded nor be readily interpreted as doublets arising from parallel parts of the same subtest. These two factors were interpreted as "content" dimensions, _i.e._, as factors presumed to depend upon content domains partially overlapping more than one of the subtests, which happened to be sampled in different proportions by the odd and even sub-subtests that were established for the purposes of the analysis. Such an interpretation of the results assumes, of course, that if certain of the subtests are factorially complex, the items of which they are constructed are relatively less complex.

The results of the previous study are particularly clear in suggesting that this state of affairs exists in the Picture Completion (PC) and Comprehension (C) subtests. It was therefore judged to be desirable to try to verify the factorial composition of PC by a factor analysis of the items themselves and, if possible, to arrive at a relatively more meaningful separation of the items involved than that afforded by the arbitrary division according to odd and even numbers. Such results could then be used as a basis for a more definite differential interpretation of the factors and

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[1] This research was supported by the Society for the Investigation of Human Ecology.

[2] This report may be regarded as the second in a series (6) that will be concerned with the general question of relationships between aptitude and achievement tests, on the one hand, and various measures of personality on the other. The Wechsler provides a useful starting point for studies in this area since it appears to span a number of representative ability dimensions, and is widely employed clinically in the understanding of personality disorders. This report contributes to the overall series by providing additional information about three of the dimensions measured by the Wechsler.

for constructing relatively uncorrelated keys for the factors. It is the purpose of this report to describe the procedure and results obtained in such an analysis, and some of their implications.

Procedure.

The sample of subjects used was the same as that used previously (6), comprising a combined group of 228 male college and college-preparatory students. Separate scores for 22 items were obtained from the original WAIS Record Forms, and their tetrachoric intercorrelations were determined using an IBM 650 approximation procedure developed by Tucker (8). The first 21 items were the regular WAIS PC items, administered in the normal sequence according to the standard directions. (Response times were also recorded for one part of the sample, and found to be of little value in a subsequent attempt to use them.) The 22nd score was derived from the "Fishing" item of the Picture Arrangement (PA) subtest, which was counted right for this purpose whenever cards E, F, G, and H were in the correct relative sequence. (Interchanges of cards E and G, or F and G, or F and H, etc., have never been counted by us as errors for purposes of scoring PA.)

Two of the 22 items were successfully passed by such a high proportion of the sample (226 out of 228 cases) that most of their intercorrelations with other variables were flagged by the computer as unreliable approximations. These items, numbers 1 and 3, were therefore dropped from the analysis. The correlation matrix obtained for the remaining 20 items is shown in Table 1, together with the proportion of the total sample getting each item right.

The factor analysis itself was carried out by an iterative procedure, employing initial communality estimates of zero for all variables in order to restrict as far as possible the appearance of specific factors in the common factor space. In each iteration the complete set of latent roots and principal axis factors was determined, and those factors were regarded as significant whose latent roots were positive and greater in magnitude than the largest negative latent root. Using this rule-of-thumb procedure it became clearly evident after the second iteration that the correlations

Table 1.

TETRACHORIC INTERCORRELATION MATRIX (N=228)

Item	2	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	PA	P_i
2	--	30	-14	30	23	19	08	46	-32	18	18	57	10	00	39	24	29	12	06	17	.95
4	30	--	26	29	10	36	29	21	06	06	05	24	19	-08	16	07	14	20	23	31	.89
5	-14	26	--	10	21	-13	15	18	15	11	15	22	06	20	22	15	-17	00	02	10	.85
6	30	29	10	--	10	42	29	15	06	06	16	30	-03	06	16	21	14	31	23	25	.89
7	23	10	21	10	--	26	29	-01	05	11	35	16	17	05	-01	18	06	-06	41	-16	.91
8	19	36	-13	42	26	--	71	32	15	23	35	06	39	05	14	10	27	34	00	00	.84
9	08	29	15	29	29	71	--	26	19	35	29	-04	47	15	24	34	15	11	26	10	.78
10	46	21	18	15	-01	32	26	--	00	-04	34	17	-06	-13	24	02	19	27	13	03	.74
11	-32	06	15	06	05	15	19	00	--	06	12	-12	19	20	11	13	05	22	11	02	.73
12	18	06	11	06	11	23	35	-04	06	--	26	06	23	29	02	42	-13	42	-16	18	.76
13	18	05	15	16	35	35	29	34	12	26	--	-13	18	-02	21	12	21	11	-09	04	.90
14	57	24	22	30	16	06	-04	17	-12	06	-13	--	04	-08	22	19	15	04	11	04	.66
15	10	19	06	-03	17	39	47	-06	19	23	18	04	--	27	31	19	-14	-11	06	15	.79
16	00	-08	20	06	05	05	15	-13	20	29	-02	-08	27	--	11	49	05	12	26	13	.73
17	39	16	22	16	-01	14	24	24	11	02	21	22	31	11	--	32	19	13	11	19	.49
18	24	07	15	21	18	10	34	02	13	42	12	19	19	49	32	--	-17	43	28	03	.80
19	29	14	-17	14	06	27	15	19	05	-13	21	15	-14	05	19	-17	--	03	-01	14	.40
20	12	20	00	31	-06	34	11	27	22	42	11	04	-11	12	13	43	03	--	18	02	.56
21	06	23	02	23	41	00	26	13	11	-16	-09	11	06	26	11	28	-01	18	--	00	.22
PA	17	31	10	25	-16	00	10	03	02	18	04	04	15	13	19	03	14	02	00	--	.59

would support just three factors; two further iterations were carried out in order to stabilize the communality estimates for this number of factors. The resulting factor matrix and communalities are shown in Table 2.

Since there are only three factors, and since all of the items have appreciable positive loadings on the first factor, the easiest way to examine the results is in the extended vector representation (7) of Figure 1. With such a representation, if there is a simple structure, it will appear that most of the points can be regarded as falling along the edges and particularly at the corners of a triangle; the corners of the triangle then mark the extensions of the factor axes.

The raw quartimax factor axes were obtained next and are shown in Figure 1 at the corners of the triangular configuration which has been drawn with broken lines. These axes have been arbitrarily designated as I, II, and III. In view of the closeness of items 8, 16, and 14 to the respective corners of the raw quartimax configuration, and the near perfect mutual orthogonality of these items, it was decided to use these particular items to anchor the final rotation. The numerical loadings computed on this basis (5) are reported in Table 3.

Estimates were obtained of the relative positions within the total configuration of the PC scores employed in the previous analysis (6), which had been arbitrarily based on the odd- and even-numbered items separately. These estimates were computed by weighting the direction cosines of the constituent items in proportion to $h[p_i(1-p_i)]$, where p_i is the proportion of the sample answering item i correctly and h is the square root of the item's communality. These scores are represented in the figure by the symbols PC_O and PC_E, respectively. These are not included in Table 3, however, since we cannot readily estimate the communalities of PC_O and PC_E.

Discussion.

Only items 11 and 19 appear to represent appreciable violations of the positive manifold bounded by the hyperplanes of the rotated factors. Since both of these items are of comparatively low communality, this may be due

Table 2.

UNROTATED FACTOR MATRIX

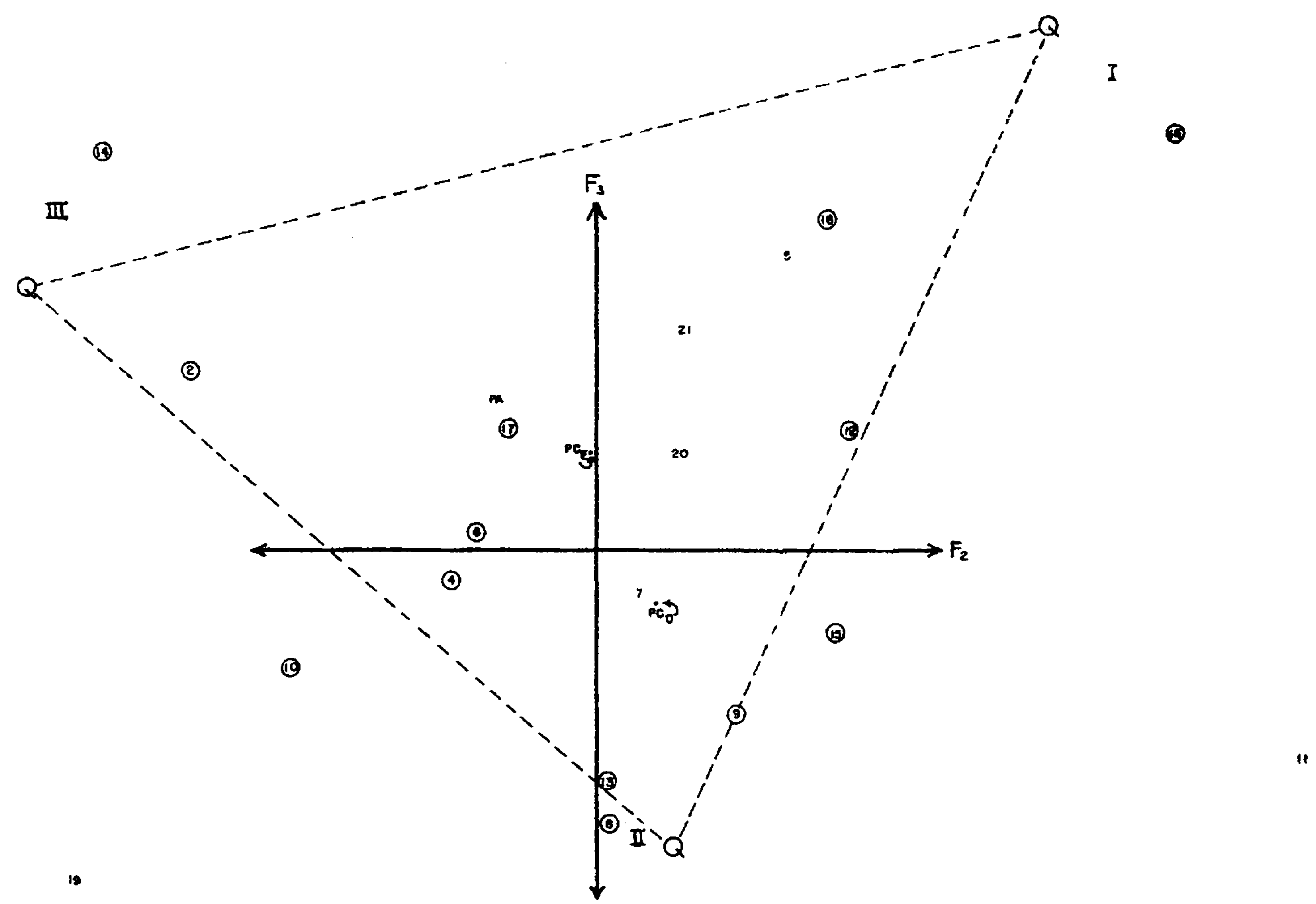
Table 3.

ROTATED FACTOR MATRIX

Item	F_1	F_2	F_3	h^2	I	II	III
2	525	-621	265	732	-090	232	818
4	448	-188	-037	238	029	370	317
5	204	113	174	084	262	054	115
6	482	-165	028	261	095	357	353
7	332	039	-039	114	144	285	106
8	685	019	-533	754	007	868	006
9	708	280	-336	693	313	770	-051
10	400	-352	-130	301	-162	387	354
11	176	360	-106	172	277	212	-224
12	393	286	127	252	441	235	058
13	402	011	-267	233	029	481	030
14	315	-449	358	429	-001	015	655
15	392	272	-088	236	312	368	-054
16	265	442	318	366	605	020	-001
17	440	-108	152	229	187	247	364
18	539	361	513	685	767	110	291
19	217	-323	-204	193	-257	290	208
20	394	093	109	176	291	243	181
21	279	073	175	114	264	111	178
PA	214	-062	090	058	093	111	193
Σa^2	3.475	1.612	1.232	6.320	1.819	2.607	1.895

Figure 1.

EXTENDED VECTOR REPRESENTATION



to unreliability in the determination of the directions of their vectors. Aside from the locations of these two items, the simple structure is judged to be obvious in the data, and the positions of the rotated axes are judged to be satisfactory as a means of describing this structure. It is appropriate for us to consider what interpretations are available for the three factors.

1. Rapaport (4, p. 230ff.) provides the most complete rationale available for PC as a whole, and concludes that "the function underlying achievement on Picture Completion is concentration acting upon visually-perceived material." Rapaport's "concentration" is not different from reasoning, but is the "kernal of all reasoning." On the other hand, "concentration" is not mere effortless attention and is not merely another type of information. While it may be that some function such as concentration underlies all three of our factors, our analysis emphasizes the existence of three relatively uncorrelated varieties of PC item for which correspondingly distinct rationales should be furnished.

Rapaport does go on to recognize three causes of failure on PC items, even though each of them is seen as interfering with the central and essential functioning of concentration. If Rapaport's observations are valid, it may be possible to relate these three kinds of failure to our three factors. However, we must remember that his observations and discussion are directed primarily at understanding "abnormal" processes, whereas our factor analysis is based on a group of essentially "normal" subjects. The dominant mechanisms underlying success and failure on PC items may be quite different in normal and abnormal groups. Furthermore, we must remember that his discussion is based on WB-I, whereas we are dealing with a somewhat different set of items in the WAIS.

One type of failure results when a "query for information replaces concentration," and is illustrated by Rapaport in terms of items 2, 10, and 14.[3]

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[3] These are the WAIS item numbers, which we shall use consistently to avoid confusion within the present context. In WB-I these were items 6, 13, and 7, respectively.

It is notable that these are precisely the three items with the highest loadings on our Factor III. Among items 4, 6, and 17, which correlate with Factor III almost as well as item 10 does, items 4 and 17 were not even present in WB-I. It would appear that here we have more than a mere chance coincidence, and that low scores on Factor III may arise in subjects who feel uncertain about the reliability of the information they would like to apply to the solution of these items. It seems to us that the awareness of uncertainty is primarily predisposing to failure on this factor, since the subject may actually suspect the correct answer without having the confidence to offer it as an overt guess.

A second type of failure results from what Rapaport terms an "increase of distance from the picture," or "impaired contact with reality." The only item used to illustrate this phenomenon is item 18, and this is the item with the highest loading on our Factor I! However, our sample of normal subjects provided no dramatic expressions of "loss of reality testing" such as Rapaport refers to, and we must be careful lest we overinterpret the coincidence of a single item.

The third type of failure results from what Rapaport terms "loss of distance," when the subject considers as missing something that was never intended to be in the drawing but which the subject would have to have there in a corresponding real situation. The items that Rapaport employs to illustrate this phenomenon have not segregated themselves in our factor analysis. However, even our sample of normal subjects has given an appreciable number of responses that appear to be of this type. Thus, 48 of our 228 subjects say an oarsman is missing from the rowboat, 35 say a rider is missing from the horse, 31 say a bow is missing from the violin, and 25 say a hand is missing from the water-pitcher. In view of the appreciable frequencies of these responses we had scored our subjects for them, and had included them when the item intercorrelations were run; they could not be used in the factor analysis because they are counted "wrong," and we had already included the "right" answers to the same items. The intercorrelations of these four responses are as follows:

		6	8	9	19
(Hand)	6	-			
(Bow)	8	.39	-		
(Oarsman)	9	.27	.79	-	
(Rider)	19	.19	.61	.41	-

Comparison of these values with those reported for the same items in Table 1 shows that the "wrong" responses form a comparatively tighter cluster than the "right" responses--principally as a result of item 19 coming more nearly into line with items 8 and 9. Examination of Table 3 discloses that even the "right" responses for these items correlate with Factor II more highly than with either of the other factors.

In order to test the relationship of "loss of distance" to Factor II, we computed and correlated two scores for our sample. The "loss of distance" score was based on the indicated "wrong" responses to items 6, 8, 9, and 19, and had an estimated reliability of 0.74. The Factor II score was based on the "right" responses to items 8, 9, 13, and 15, and had an estimated reliability of 0.62. The intercorrelation of these two scores was found to be -0.69, which becomes -1.01 upon correction for attenuation. It is evident from this result that we may regard Factor II as "maintenance of distance," which is the obverse of Rapaport's "loss of distance."

This result provides indirect additional support for the identification of our Factor I as "maintenance of contact," which is the obverse of Rapaport's "loss of contact." We would further conclude that our factorial results provide substantial confirmation for Rapaport's discussion of the causes of failure on PC items, while having no direct bearing on the hypothesis that "concentration" underlies all of the items.

2. Based on the previous factor analysis of split-half subtest scores using the same sample of cases (6), we had anticipated the emergence of two factors in the present analysis, and had hypothesized that these would

reflect a distinction "between responses that depend on fairly specific prior experience (i.e., learning) and other responses that depend solely on noticing the relevant stimulus information in the PC item (i.e., recognition of incongruity)." The latter function was attributed to PC_E more than to PC_O, while the former was attributed more to PC_O than to PC_E.

The emergence of three factors poses a complicated problem in interpretation. Which factors, if any, correspond to the hypothesized functions? How should we treat the factor or factors left over? Why did the previous analysis fail to anticipate that three factors would emerge? It is simplest to begin with the last question.

It can be argued that the previous analysis did anticipate the involvement of PC in three factors, but that we were led astray in the interpretation of those results by our desire to be conservative in judging the statistical significance of the factors. If we were to accept Factor XI of the previous analysis as significant, and were to rotate it slightly in the plane of Factor VI so as to place PC_O in the hyperplane of Factor XI, PC_E would acquire a loading of about +0.3. Such a loading is of typical magnitude for the last three factors of the previous study, and is comparable with the loading of PC_O on Factor IX. Had it not been necessary to question the statistical significance of Factor XI, it would have been natural to implicate Factor XI along with Factors VI and IX in accounting for total scores on PC.

If we assume that we should have hypothesized three factors instead of two, we may utilize the positions of PC_O and PC_E in the configuration of Figure 1 to tell us which factor is which. Factor II is much closer to PC_O than to PC_E, and appears to correspond to Factor IX of the previous analysis. Factor III is much closer to PC_E than to PC_O, and appears to correspond to Factor XI of the previous analysis. Factor I is slightly closer to PC_E than to PC_O, and appears to correspond to Factor VI of the previous analysis. Except for Factor II, the interpretations that are suggested by our previous discussion (6) appear to be generally applicable to the items correlated with the factors.

With respect to Factor II, it might still be argued that correct solutions for any of the items correlated with the factor are at least facilitated by prior knowledge of the objects involved, that the objects involved are such that important proportions of a group of subjects will not be intimately acquainted with them, and that, in fact, proof of the correct solution can only be based on prior knowledge or experience. While this argument would support the identification of Factor II as "learning" or "breadth of experience," we are inclined not to stress this aspect of the factor in view of the obvious relevance of experience in Factor III, which we have noted above.

In another way of approaching Factor II, it can be argued that in every case the subject is required to recognize a missing part that is needed to complete even that portion of an object that is presented. This would also be true in Factor III. The difference between Factor II and Factor III would then reside in the fact that in Factor II it is obvious what is missing, either because it is like something else that is present in the picture or because the object is extremely familiar, whereas in Factor III the subject may be reasonably uncertain either as to what the object is or what sort of thing might be missing from it or both. This interpretation of Factor II appears to provide a functional definition of "maintenance of distance," which is a label we have already associated with this factor. It may also provide a usable definition to guide the choice of additional items to measure this aspect of PC performance.

With respect to Factor I, it may be seen that the subject must pay attention to incongruities in the overall stimulus configuration. Since these incongruities may be completely substantiated by the contents of the stimulus, using minimal assumptions, uncertainty and Factor III do not come into play. Since each of the isolable objects in these pictures is structurally adequate, maintenance of distance and Factor II do not come into play. Successful performance on Factor I requires the subject to be aware of functional disharmony involving objects related in his environment. This interpretation appears to provide a relatively more operational

definition for the notion of "maintenance of contact" which we have already associated with this factor.

3. Although it is eminently clear from the discussion up to this point that PC may not be safely regarded as a homogeneous set of items, measuring only one function and varying only in difficulty, this implication of our results will bear explicit statement. The finding that PC is definitely multidimensional, regardless of the rotation or interpretation of the dimensions, provides a sufficient explanation of the relatively marked discrepancies that have been noted in efforts to place the PC items along a single scale of difficulty (1, 2, 3).

4. An important assumption implicit in much of our discussion up to this point is that there are not more than three factors operating in the PC subtest. The existence of any fourth factor would cast doubt upon the propriety of our factor identifications based on Rapaport's rationale, and upon the matching of these factors with those of the previous analysis. Support for the sufficiency of three factors comes to some extent from the apparent identity of the factors with Rapaport's antecedent discussion (4), but primarily from the factor-analytic procedure itself, which was unusually unambiguous in indicating the adequacy of three common factors.

Additional verification may be sought by investigating two predictions based upon the proposed matching of factors between the two factor-analytic studies. If Factor II is the same as the previous Factor IX, then certain Comprehension items--particularly odd-numbered C items--should correlate with the items marking Factor II. If Factor III is the same as the previous Factor XI, and our previous identification of that factor is correct, then Object Assembly items should correlate with Factor III items; OA items should not be related to Factor I or Factor II. Factor VI of the previous study, which we have matched with Factor I, does not involve any other subtest sufficiently to permit a similar parallel prediction. Therefore, if the two possible predictions are sustained, Factor I will be regarded as the unique contribution of the PC subtest to the Wechsler Battery.

Table 4 presents tetrachoric correlations of the four OA items with the three PC items that appear both from the numerical results and the discussion to be the best representatives for the three factors. That there is a significant tendency for the largest values to appear in connection with Factor III may be shown in a variety of ways, among which the reader may choose for himself. The results are particularly clear for OA item #3, the hand, which would normally be regarded as the best OA item. The results are least clear for OA item #4, the elephant, which was added to WB-I in building the WAIS.[4]

While these results might simply be accepted as supporting one of the two predictions, it is of interest to consider how OA comes to be related to Factor III. Rapaport discusses OA along with BD and DS under the heading of visual-motor coordination (4). Certainly, the most obvious empirical fact involving OA is its high correlation with BD, this being typically the highest correlation between any pair of subtests. However, this relationship is accounted for by Factor I of the previous study (6), on which both BD and OA had high loadings; we are here concerned with Factor XI of the previous results, which we have now matched with the present Factor III. A major difference between BD and OA is that in the latter the subject is not told what the pieces will make when they have been put together, whereas in the former he is given a design to copy. In other words, OA may be considered to be simply BD plus "uncertainty." Subjects who are accustomed to knowing what they are about are actually at a relative disadvantage in the OA task, for any time they may spend seeking insight or otherwise fretting about how silly they must look to the examiner because

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[4] The reader may reasonably ask at this point why we did not include the OA and C items in our factor analysis in the first place. Had we anticipated the relevance of OA we would certainly have included these four items. We deliberately excluded the Comprehension items because there would otherwise have been too many variables for handling by the computer programs available to us for this study. A similar study that will focus its attention on Comprehension is planned.

Table 4.

OA vs PC ITEM INTERCORRELATIONS

	OA#1	OA#2	OA#3	OA#4	
PC #18	22	11	19	28	I
16	15	13	10	12	
12	-20	00	00	07	
PC # 8	-04	17	17	21	II
9	-16	12	15	25	
13	29	39	13	30	
PC # 2	21	54	26	23	III
14	22	11	31	07	
10	46	42	29	27	

Table 5

C vs PC ITEM INTERCORRELATIONS

	C# 5	C# 6	C#11	C#13	
PC #18	-10	19	-30	-17	I
16	03	03	-03	01	
12	05	08	-06	03	
PC # 8	16	40	26	20	II
9	22	21	21	06	
13	02	33	15	17	
PC # 2	09	14	-33	34	III
14	14	05	-14	00	
10	06	14	16	12	